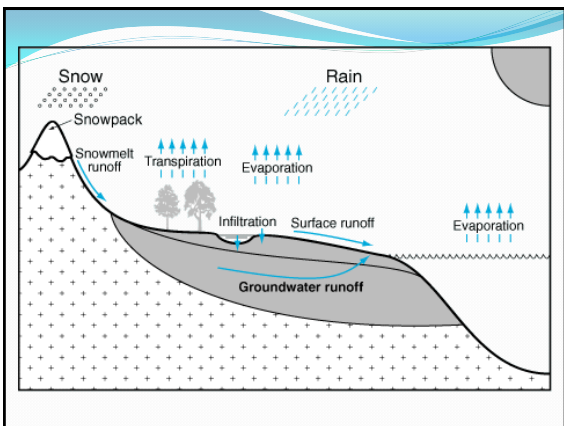


Hydrological modelling

GG22A: GEOSPHERE & HYDROSPHERE
Hydrology

Hydrological modelling

- Computer modelling has become an integral part of the decision making process for water engineers and managers
 - Model results are increasingly used as justification for infrastructure development (flood defences)
 - Practicalities of applying a computer model very much easier than previously - false confidence.

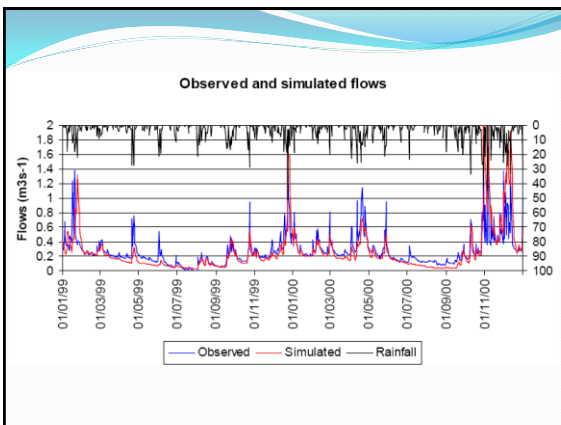


Hydrological cycle

- Water recycling processes link water in the atmosphere, on the continents, and in the oceans
 - Models aim to represent these processes
 - Simplified representations of reality
 - Models used to test hypotheses or make predictions

Parameters, calibration, validation

- *parameters*: constants that define model characteristics, but vary between applications
- *calibration*: estimation of model parameters by comparing observed with predicted
 - objective function
- *validation*: verification of model fit against independent data



Model calibration

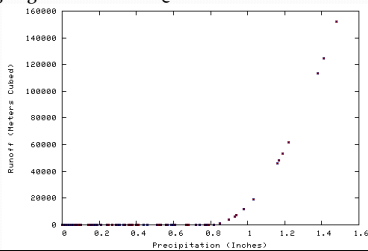
- Major problems:
 - different criteria for goodness of fit
 - different parameter combinations can give similar model output and similar quality fit (equifinality)

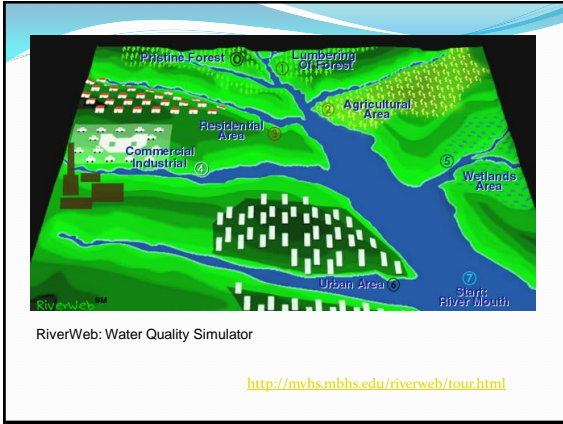
Types of model

- stochastic vs deterministic
 - Stochastic: Simulate a random sequence of numbers with statistical properties similar to those of real data
 - Deterministic: model the transformation of input to output

Types of model

- Empirical
 - e.g. regression model $Q = aP + b$





Types of model

- Conceptual model
 - Treat catchment as a series of stores with fluxes between them
 - Parameters determine size of store and rate of flows

Mass Balance

Forms the basis to most hydrological and hydrochemical models

$$M(t + \Delta t) - M(t) = \left(\frac{I'(t) + I'(t + \Delta t)}{2} \right) \Delta t - \left(\frac{O'(t) + O'(t + \Delta t)}{2} \right) \Delta t$$

Average Input Rate Average Output Rate

$$\frac{M(t + \Delta t) - M(t)}{\Delta t} = \left(\frac{I'(t) + I'(t + \Delta t)}{2} \right) - \left(\frac{O'(t) + O'(t + \Delta t)}{2} \right)$$

$$\frac{dM}{dt} = \lim_{\Delta t \rightarrow 0} \frac{M(t + \Delta t) - M(t)}{\Delta t}$$

$$= \left(\frac{2I'(t)}{2} \right) - \left(\frac{2O'(t)}{2} \right)$$

$$= I'(t) - O'(t)$$

Assuming constant density:

$$\frac{dV}{dt} = I(t) - O(t)$$

V = volume of water within the control volume [L³]

I = volume inflow rate [L³ T⁻¹]

O = volume outflow rate [L³ T⁻¹]

What is a typical control volume?

We can treat the land phase of the hydrological cycle as comprising one compartment. For an arbitrary area of land need to identify inputs and outputs:

p is the precipitation rate

r_{si} is the surface water inflow rate

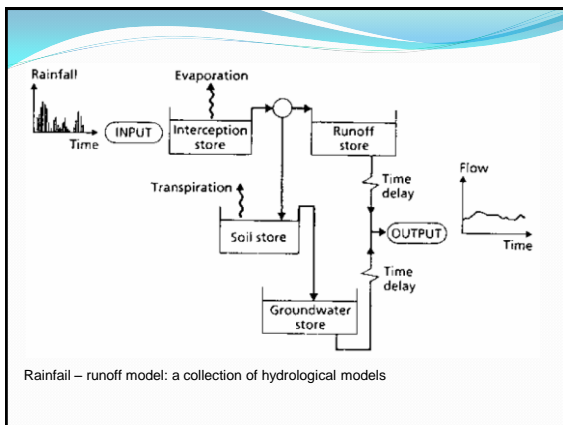
r_{so} is the surface water outflow rate

r_{gi} is the groundwater inflow rate

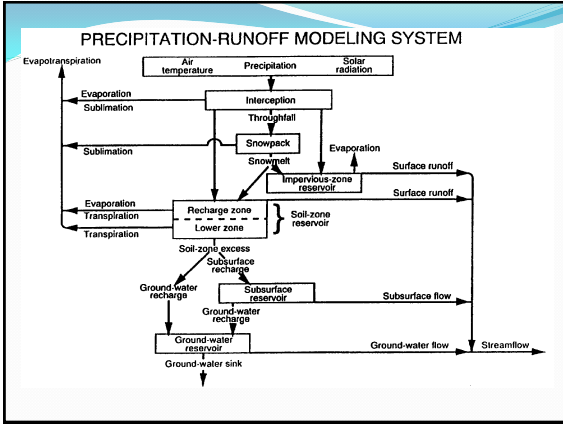
r_{go} is the groundwater outflow rate

et is the evapotranspiration

$$\frac{dV}{dt} = p + r_{si} + r_{gi} - r_{so} - r_{go} - et$$



Rainfall – runoff model: a collection of hydrological models

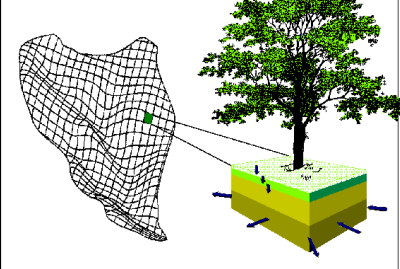


Conceptual rainfall-runoff models

- Physically meaningful parameters
- Temporal and spatial variability
 - due to temporal and spatial distribution of precipitation and other properties of the catchment

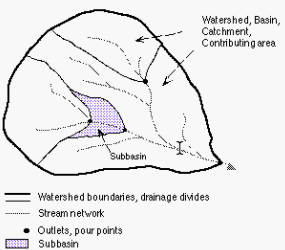
Types of model

- Lumped vs. distributed
 - Lumped: treat catchment as one single unit
 - Distributed: treat catchment as multiple units
 - A catchment is then defined as all points that potentially can contribute surface water to a particular river station.
 - The topography of the land surface usually controls where divides are drawn.



Distributed models incorporate the effects of topography through direct use of the digital elevation data during computation, along with process-level knowledge.

- **Drainage system** - The area upon which water falls and the network through which it travels to an outlet.
- **Catchments** - an area that drains water and other substances to a common outlet as concentrated flow (watersheds, drainage basin, contributing area)
- **Subbasin** - That upstream area flowing to an outlet as overland flow
- **Pour Point** - A location at which the contributing area can be determined.
- **Drainage Divide** - The boundary between two basins. This is an area of divergent flow.



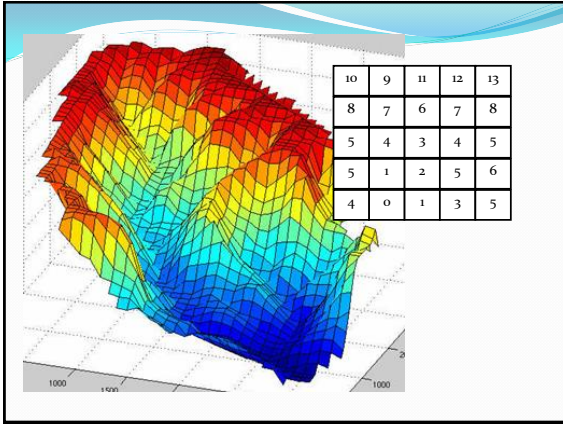
Watershed, Basin, Catchment, Contributing area

Subbasin

— Watershed boundaries, drainage divides
 - - - Stream network
 • Outlets, pour points
 ■ Subbasin

GIS and hydrological modelling

- GIS can automate many tasks required in hydrological modelling
 - e.g. location of drainage divides
 - Fast and accurate



Slope

- Usually calculated on a 3x3 window with the center cell being the target cell.
- Slope is calculated from the center cell to each of the 8 downhill neighbours
- Greatest slope is assigned to the center cell

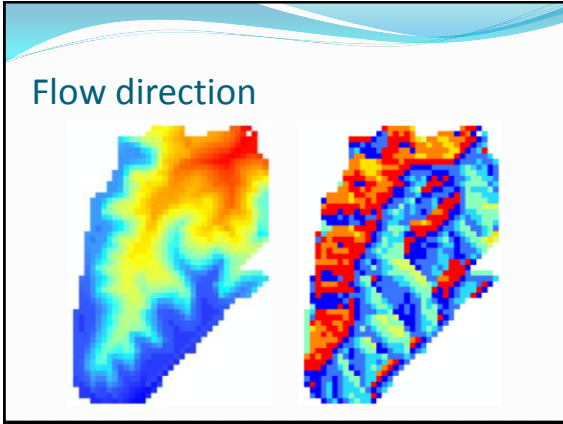
10	9	11	12	13
8	7	6	7	8
5	4	3	4	5
5	2	1	5	6
4	1	0	3	5

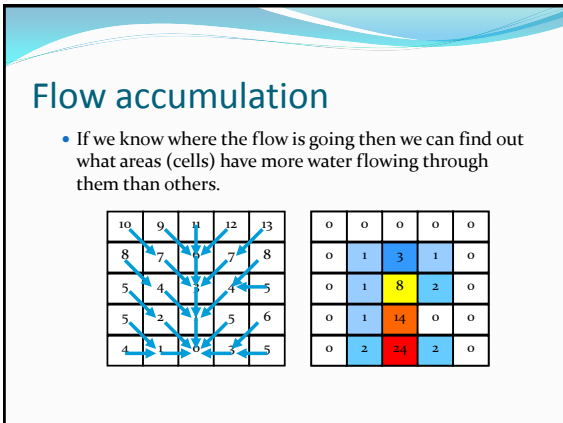
-3	-3	-5	-6	-6
-4	-4	-3	-4	-4
-3	-3	-2	-3	-1
-4	-2	-1	-5	-3
-3	-1	0	-3	-2

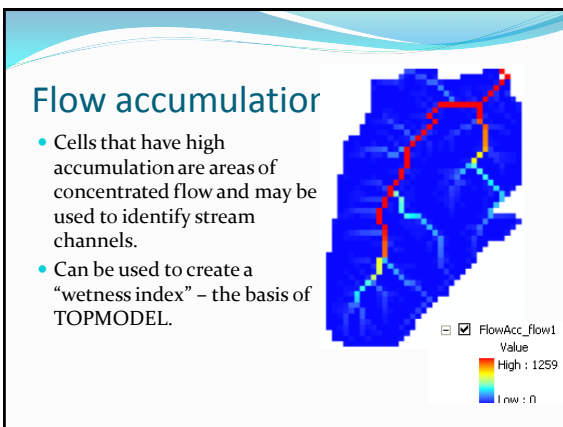
Flow direction

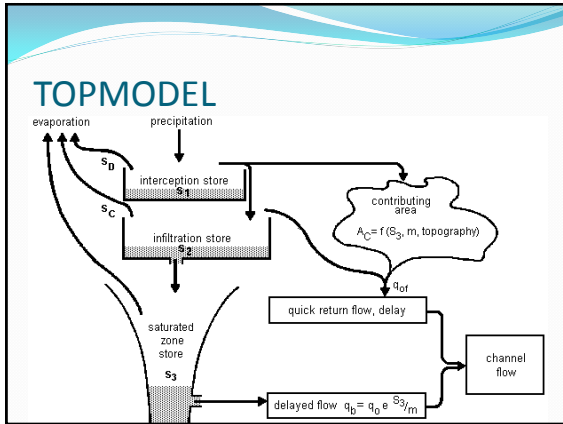
- Assigned to the downslope cell with steepest slope

10	9	11	12	13
8	7	6	7	8
5	4	3	4	5
5	2	1	5	6
4	1	0	3	5









TOPMODEL

- Three stores are used in the model: interception, infiltration and groundwater.
- Evaporation represents loss from the system.
- It is assumed that water leaves the catchment by either a quick (qof) or delayed (qb) route.
 - Flow is always by the delayed route unless rainfall exceeds infiltration capacity, or falls on already saturated land (Beven et al., 1984).

TOPMODEL

- Delayed flow:

$$q_b = q_0 \exp S_3/m$$
 - q_b = flow reaching channel from the store
 - S_3 = saturated zone store
 - m = subsurface flow parameter
 - q_0 = subsurface flow when $S_3 = 0$

Delayed flow

- Determines the amount of flow in the channel during dry spells.
 - S_3 is exponential, and is zero when the average soil water content over the basin is just saturated (Beven and Kirkby, 1979).
 - Positive values of S_3 represent a moisture surplus and negative values a deficit (below the average across the basin).

Contributing area & quick flow

- $AC = f(S_3, m, \text{topography})$

$S_3 =$ saturated zone store
 $m =$ subsurface flow parameter

- Quick flow incorporates a high degree of spatial resolution into the model, with the use of the contributing area (AC) in its calculation

Quick flow

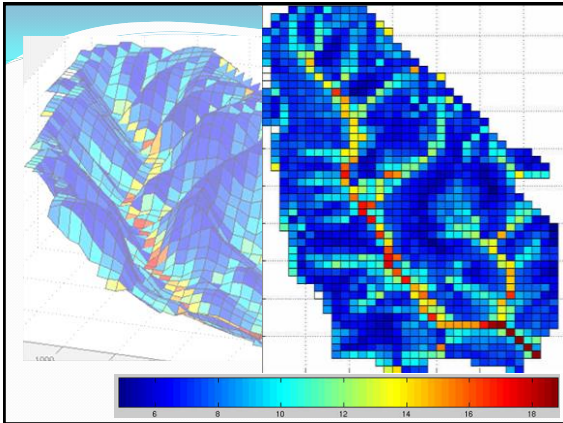
- Quick flow (overland flow) is dependent on conditions of the saturated zone store (S_3) and the subsurface flow parameter (m).
 - These two variables are fundamental to the model.
- They are combined with topography with the use of a topographic index.
 - This calculates the likelihood of saturation at each point in the catchment with the use of a raster Digital Terrain Model.

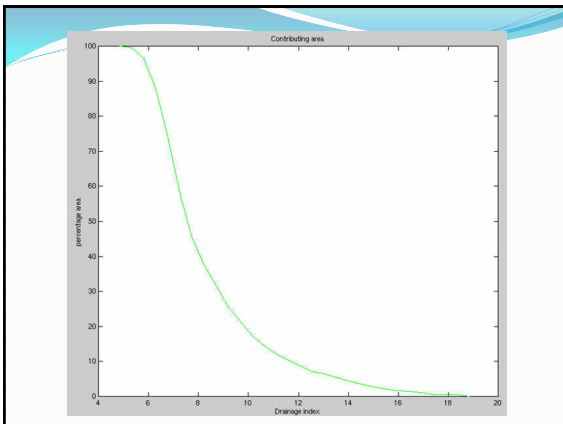
Topographic saturation index

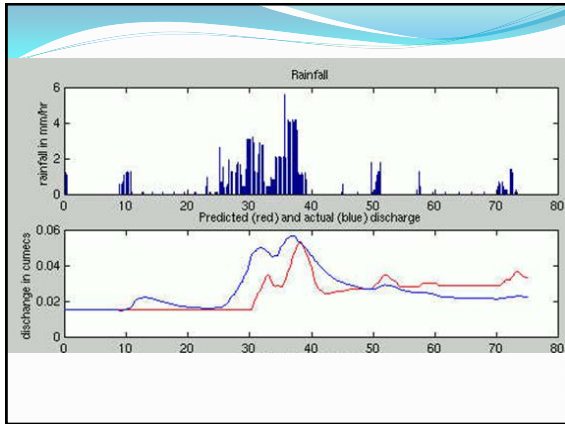
$$k = \ln(a / \tan \beta)$$

a = area draining through point x, y from upslope.
 tan β = local slope angle at point x, y.

- Thus, points in the catchment with large areas upslope, and points of low slope angle are more likely to become saturated, and will have a larger saturation index.
- Pixels having the same or similar saturation indexes are assumed to behave in a hydrologically similar manner.







Problems of sink cells

- Topographic index only works if a downslope direction can be calculated for all cells
- “Sink cells” are cells of internal drainage: i.e. no outflow:

River flow forecasting

- Rainfall-runoff models may be used in real-time forecasting mode
 - Utilise additional input information in the form of recently measured outflow data
 - The LISFLOOD system

